On possible *a-priori* “imprinting” of General Relativity itself on the performed Lense-Thirring tests with LAGEOS satellites

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Abstract

The impact of possible a-priori “imprinting” effects of general relativity itself on recent attempts to measure the Lense-Thirring precessions with the LAGEOS satellites orbiting the Earth and the terrestrial geopotential models by the dedicated mission GRACE is investigated. It is analytically shown that general relativity, not explicitly solved for in the GRACE-based models, may “imprint” their even zonal harmonic coefficients J_ℓ at a non-negligible level, given the present-day accuracy in recovering them. This translates into a bias of the LAGEOS-based relativistic tests as large as the Lense-Thirring effect itself. Further analyses should include general relativity itself in the GRACE data processing by explicitly solving for it.

Keywords: Experimental studies of gravity; Satellite orbits; Harmonics of the gravity potential field
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1 Introduction

The term “gravitomagnetism” [1, 2, 3] (GM) denotes those gravitational phenomena concerning orbiting test particles, precessing gyroscopes, moving clocks and atoms and propagating electromagnetic waves [4, 5] which, in the framework of the Einstein's General Theory of Relativity (GTR), arise from non-static distributions of matter and energy. In the weak-field and slow motion approximation, the Einstein field equations of GTR, which is a highly non-linear Lorentz-covariant tensor theory of gravitation, get linearized [6], thus looking like the Maxwellian equations of electromagntism. As a consequence, a “gravitomagnetic” field $\vec{B}_g$, induced by the off-diagonal components $g_{0i},\ i = 1, 2, 3$ of the space-time metric tensor related to mass-energy currents, arises. In particular, far from a localized rotating body

with angular momentum $\vec{S}$ the gravitomagnetic field can be written as [7]

$$\vec{B}_g(\vec{r}) = \frac{G}{cr^3}\left[\vec{S} - 3\left(\vec{S}\cdot\hat{r}\right)\hat{r}\right], \tag{1}$$

where G is the Newtonian gravitational constant and c is the speed of light in vacuum. It affects, e.g., a test particle moving with velocity $\vec{v}$ with a non-central acceleration [7]

$$\vec{A}_{\rm GM} = \left(\frac{\vec{v}}{c}\right)\times\vec{B}_g. \tag{2}$$

It is the cause of the so-called Lense-Thirring[1] effect [9], which is one of the most famous and empirically investigated GM features; another one is the gyroscope precession [10, 11], goal of the Gravity Probe B (GP-B) mission [12] whose data analysis is still ongoing [13].

The Lense-Thirring effect consists of small secular precessions of the longitude of the ascending node Ω and the argument of pericenter ω of the orbit of a test particle in geodesic motion around a slowly rotating body with angular momentum $\vec{S}$; they are

$$\dot\Omega_{\rm LT} = \frac{2GS}{c^2a^3(1-e^2)^{3/2}},\ \dot\omega_{\rm LT} = -\frac{6GS\cos I}{c^2a^3(1-e^2)^{3/2}}, \tag{3}$$

where a is the semimajor axis of the satellite's orbit, e is its eccentricity and I is the inclination of the orbital plane to the equatorial plane of the central body.

Concerning the possibilities of measuring it in the terrestrial gravitational field, soon after the dawn of the space age with the launch of Sputnik in 1957 it was proposed by Soviet scientists to directly test the Lense-Thirring effect with artificial satellites orbiting the Earth. In particular, V.L. Ginzburg [14, 15, 16] proposed to use the perigee of a terrestrial spacecraft in highly elliptic orbit, while A.F. Bogorodskii [17] considered also the node. In 1977-1978 Cugusi and Proverbio [18, 19] suggested to use the passive geodetic satellite LAGEOS, in orbit around the Earth since 1976 and tracked with the Satellite Laser Ranging (SLR) technique, along with the other existing laser-ranged targets to measure the Lense-Thirring node precession. Since such earlier studies it was known that a major source of systematic error is represented by the fact that the even ($\ell = 2, 4, 6, ...$) zonal ($m = 0$) harmonic coefficients $J_\ell, \ell = 2, 4, 6$ of the multipolar expansion of

[1] According to a recent historical analysis, it should be more correct to speak about an Einstein-Thirring-Lense effect [8].

the classical part of the terrestrial gravitational potential, accounting for its departures from spherical symmetry due to the Earth's diurnal rotation, induce competing secular precessions of the node and the perigee of satellites [20] whose nominal sizes are several orders of magnitude larger than the Lense-Thirring ones. In the case of the node, the largest precession is due to the first even zonal harmonic J_2

$$\dot{\Omega}_{J_2} = -\frac{3}{2} n \left(\frac{R_\oplus}{a}\right)^2 \frac{\cos I J_2}{(1-e^2)^2}, \tag{4}$$

where $R_\oplus$ is the Earth's mean equatorial radius and $n \doteq \sqrt{GM_\oplus/a^3}$ is the satellite's Keplerian mean motion. For the other higher degrees the analytical expressions are more involved; since they have already been published in, e.g., Ref. [21], we will not show them here.

Tests have started to be effectively performed about 15 years ago by Ciufolini and coworkers [22] with the LAGEOS and LAGEOS II satellites[2], according to a strategy by Ciufolini [23] involving the use of a suitable linear combination of the nodes Ω of both satellites and the perigee ω of LAGEOS II in order to remove the impact of the first two multipoles of the non-spherical gravitational potential of the Earth. Latest tests have been reported by Ciufolini and Pavlis [24, 25], Lucchesi [26] and Ries and coworkers [27] with only the nodes of both the satellites according to a combination of them explicitly proposed by Iorio[3] [28]. The total uncertainty reached is still matter of debate [32, 33, 34, 35, 36, 37, 38] because of the lingering uncertainties in the Earth's multipoles and in how to evaluate their biasing impact; it may be as large as $\approx 20-30\%$ according to conservative evaluations [32, 35, 36, 37, 38], while more optimistic views [24, 25, 27] point towards $10-15\%$.

To be more specific, the node-only combination used in the latest tests is

$$\dot{\Omega}^{\rm LAGEOS} + c_1 \dot{\Omega}^{\rm LAGEOS\ II},\ c_1 = 0.544. \tag{5}$$

It was designed to remove the effects of the static and time-varying components of J_2, so that eq. (5) is affected by the remaining even zonals of higher degree $J_4, J_6, \ldots$. The gravitomagnetic trend given by eq. (5) amounts to 47.8 milliarcseconds year^{-1} (mas yr^{-1} in the following) since the Lense-Thirring node precessions for the LAGEOS satellites are 30.7 mas yr^{-1} (LAGEOS) and 31.5 mas yr^{-1} (LAGEOS II). The Lense-Thirring signal

[2] LAGEOS II was launched in 1992.
[3] See also Refs. [29, 30, 31].

is usually extracted from long time series of computed[4] "residuals" of the nodes of LAGEOS and LAGEOS II obtained by processing their data with a suite of dynamical force models which purposely do not encompass the gravitomagnetic force itself [39, 40]. The action of the even zonals is accounted for by using global solutions for the Earth's gravity field, in which general relativity has never been explicitly solved for[5], produced by several institutions around the world from data of dedicated satellite-based missions like GRACE[6] [42].

GRACE recovers the spherical harmonic coefficients of the geopotential from both the tracking of the two satellites by GPS and the observed intersatellite distance variations [43]. The possible "memory" effect of the gravitomagnetic force in the satellite-to-satellite tracking was preliminarily addressed in Ref. [32]. Here we will focus on the "imprint" coming from the GRACE orbits which is more important for us because it mainly resides in the low degree even zonals.

2 A-priori "imprinting" of General Relativity on the GRACE-based models

Concerning that issue, Ciufolini and Pavlis write in Ref. [33] that such a kind of leakage of the Lense-Thirring signal itself into the even zonals retrieved by GRACE is completely negligible because the GRACE satellites move along (almost) polar orbits. Indeed, for perfectly polar ($I = 90$ deg) trajectories, the gravitomagnetic force is entirely out-of-plane, while the perturbing action of the even zonals is confined to the orbital plane itself. According to Ciufolini and Pavlis [33], the deviations of the orbit of GRACE from the ideal polar orbital configuration would have negligible consequences on the "imprint" issue. In particular, they write: "the values of the even zonal harmonics determined by the GRACE orbital perturbations are substantially independent on the a priori value of the LenseThirring effect. [...] The small deviation from a polar orbit of the GRACE satellite, that is 1.7×10^{-2} rad, gives only rise, *at most*, to a very small correlation with a factor 1.7×10^{-2}". The meaning of such a statement is unclear; anyway, we will show below that such a conclusion is incorrect.

The relevant orbital parameters of GRACE are quoted in Table 1; the

[4] Actually, the nodes are not directly measurable quantities, so that speaking of "residuals" is somewhat improper.

[5] For a critical discussion of such an issue, see Ref. [41].

[6] See on the WEB http://icgem.gfz-potsdam.de/ICGEM/ICGEM.html.

Table 1: Orbital parameters of GRACE and its Lense-Thirring node precession. Variations of the orders of about 10 km in the semimajor axis a and 0.001 deg in the inclination I may occur, but it turns out that they are irrelevant in our discussion. (http://www.csr.utexas.edu/grace/ground/globe.html).

a (km)	e	I deg	$\dot{\Omega}_{\rm LT}$ (mas yr^{-1})
6835	0.001	89.02	177.4

orbital plane of GRACE is, in fact, shifted by 0.98 deg from the ideal polar configuration, and, contrary to what claimed in Ref. [33], this does matter because its classical secular node precessions are far from being negligible with respect to our issue. The impact of the Earth's gravitomagnetic force on the even zonals retrieved by GRACE can be quantitatively evaluated by computing the "effective" value[7] $\overline{C}_{\ell 0}^{\rm LT}$ of the normalized even zonal gravity coefficients which would induce classical secular node precessions for GRACE as large as those due to its Lense-Thirring effect, which is independent of the inclination I. To be more precise, $\overline{C}_{\ell 0}^{\rm LT}$ come from solving the following equation which connects the classical even zonal precession of degree ℓ $\dot{\Omega}_{J_\ell} \equiv \dot{\Omega}_{.\ell} J_\ell$ to the Lense-Thirring node precession $\dot{\Omega}_{\rm LT}$

$$\dot{\Omega}_{.\ell} J_\ell = \dot{\Omega}_{\rm LT}. \tag{6}$$

In it

$$\dot{\Omega}_{.\ell} = f(a, e, I; R_\oplus, GM_\oplus) \tag{7}$$

are the coefficients of the classical node precessions depending on the satellite's orbital parameters and on the Earth's radius and mass. Table 2 lists $\overline{C}_{\ell 0}^{\rm LT}$ for degrees $\ell = 4, 6$, which are the most effective in affecting the combination of eq. (5). Thus, the gravitomagnetic field of the Earth contributes to the value of the second even zonal of the geopotential retrieved from the orbital motions of GRACE by an amount of the order of 2×10^{-10}, while for $\ell = 6$ the imprint is one order of magnitude smaller. Given the present-day level of accuracy of the latest GRACE-based solutions, which is of the order of 10^{-12} (Table 3), effects as large as those of Table 2 cannot be neglected. Thus, we conclude that the influence of the Earth's gravitomagnetic field

[7] It must be recalled that $J_\ell = -\sqrt{2\ell + 1}\ \overline{C}_{\ell 0}$, where $\overline{C}_{\ell 0}$ are the normalized gravity coefficients.

Table 2: Effective "gravitomagnetic" normalized gravity coefficients for GRACE ($\ell = 4, 6;\ m = 0$). They have been obtained by comparing the GRACE classical node precessions to the Lense-Thirring rate. Thus, they may be viewed as a quantitative measure of the leakage of the Lense-Thirring effect itself into the second and third even zonal harmonics of the global gravity solutions from GRACE. Compare them with the much smaller calibrated errors in $\overline{C}_{40}$ and $\overline{C}_{60}$ of the GGM03S model [44] of Table 3.

$\overline{C}_{40}^{\rm LT}$	$\overline{C}_{60}^{\rm LT}$
2.23×10^{-10}	-2.3×10^{-11}

Table 3: Calibrated errors in the solved-for normalized gravity coefficients $\overline{C}_{40}$ and $\overline{C}_{60}$ according to the GGM03S global gravity solution by CSR [44]. They can be publicly retrieved at http://icgem.gfz-potsdam.de/ICGEM/ICGEM.html. Compare them with the much larger "gravitomagnetic" imprinted coefficients of Table 2.

$\sigma_{\overline{C}_{40}}$	$\sigma_{\overline{C}_{60}}$
4×10^{-12}	2×10^{-12}

on the low-degree even zonal harmonics of the global gravity solutions from GRACE may exist, falling well within the present-day level of measurability.

3 The impact of the "imprint" on the LAGEOS-LAGEOS II tests

A further, crucial step consists of evaluating the impact of such an a-priori "imprint" on the test conducted with the LAGEOS satellites and the combination of eq. (5): if the LAGEOS-LAGEOS II uncancelled combined classical geopotential precession computed with the GRACE-based a-priori "imprinted" even zonals of Table 2 is a relevant part of, or it is even larger than the combined Lense-Thirring precession, it will be demonstrated that the doubts concerning the a-priori gravitomagnetic "memory" effect are founded. It turns out that this is just the case because eq. (5)

and Table 2 yield a combined geopotential precession whose magnitude is 77.8 mas yr^{-1} (-82.9 mas yr^{-1} for $\ell = 4$ and 5.1 mas yr^{-1} for $\ell = 6$), i.e. just 1.6 times the Lense-Thirring signal itself. This means that the part of the LAGEOS-LAGEOS II uncancelled classical combined node precessions which is affected by the "imprinting" by the Lense-Thirring force through the GRACE-based geopotential's spherical harmonics is as large as the LAGEOS-LAGEOS II combined gravitomagnetic signal itself.

We, now, comment on how Ciufolini and Pavlis reach a different conclusion. They write in Ref. [33]: "However, the Lense-Thirring effect depends on the third power of the inverse of the distance from the central body, i.e., $(1/r)^3$, and the J_2, J_4, J_6... effects depend on the powers $(1/r)^{3.5}$, $(1/r)^{5.5}$, $(1/r)^{7.5}$ ... of the distance; then, since the ratio of the semimajor axes of the GRACE satellites to the LAGEOS' satellites is $\sim \frac{6780}{12270} \cong 1.8$, any conceivable "Lense-Thirring imprint" on the spherical harmonics at the GRACE altitude becomes quickly, with increasing distance, a negligible effect, especially for higher harmonics of degree $l > 4$. Therefore, any conceivable "Lense-Thirring imprint" is negligible at the LAGEOS' satellites altitude." From such statements it seems that they compare the classical GRACE precessions to the gravitomagnetic LAGEOS' ones. This is meaningless since, as we have shown, one has, first, to compare the classical and relativistic precessions of GRACE itself, with which the Earth's gravity field is solved for, and, then, compute the impact of the relativistically "imprinted" part of the GRACE-based even zonals on the combined LAGEOS nodes. These two stages have to be kept separate, with the first one which is fundamental; if different satellite(s) Y were to be used to measure the gravitomagnetic field of the Earth, the impact of the Lense-Thirring effect itself on them should be evaluated by using the "imprinted" even zonals evaluated in the first stage. Finally, in their latest statement Ciufolini and Pavlis write in Ref. [33]: "In addition, in (Ciufolini et al. 1997), it was proved with several simulations that by far the largest part of this "imprint" effect is absorbed in the by far largest coefficient J_2." Also such a statement, in the present context, has no validity since the cited work refers to a pre-GRACE era. Moreover, no quantitative details at all were explicitly released concerning the quoted simulations, so that it is not possible to judge by.

4 Conclusions

We have analytically investigated the impact of possible a-priori "imprinting" effects of GTR itself on the ongoing Lense-Thirring tests with the LA-

GEOS satellites in the gravitational field of the Earth modeled from the dedicated GRACE mission.

The classical part of the terrestrial gravitational potential, acting as a source of major systematic error because of its even zonal harmonic coefficients $\overline{C}_{\ell 0}$, is retrieved from the data of the dedicated satellite-based GRACE mission. GTR, not explicitly solved for so far in GRACE data analyses, may impact the retrieved even zonals of the GRACE models at a non-negligible level ($\approx 10^{-10} - 10^{-11}$ for $\ell = 4, 6$), given the present-day level of accuracy ($\approx 10^{-12}$ for $\ell = 4, 6$). It turns out that the resulting a-priori "imprint" of the Lense-Thirring effect itself on the LAGEOS-LAGEOS II data analysis performed to test it is of the same order of magnitude of the general relativistic signal itself.

Further, more robust tests should rely upon Earth gravity models in which GTR is explicitly solved for.

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